OCR
Oxford Cambridge and RSA

Thursday 20 June 2024 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

1336236120

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{1}{2}\theta^2,\ \tan\theta \approx \theta$ where θ is measured in radians

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Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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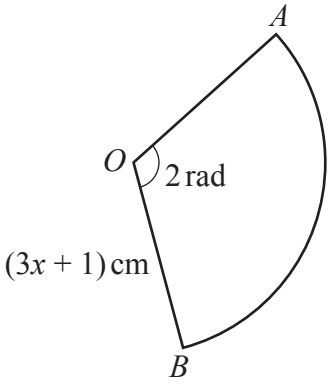
Section A
Pure Mathematics

1 Simplify each of the following.

(a) $(2a^2)^3 \times \frac{3}{4}a^{-1}$ **[2]**

(b) $\dfrac{4x^2-9}{(2x^2+5x-12)(2x+3)}$ **[2]**

2 **In this question you must show detailed reasoning.**



The diagram shows a sector AOB of a circle with centre O and radius $(3x+1)\,\text{cm}$. The angle AOB is 2 radians. The area of sector AOB is less than $(44x-7)\,\text{cm}^2$.

Find the set of possible values of x. Give your answer in set notation. **[5]**

3 **(a)** Expand $(3-2x)^{-2}$ in ascending powers of x up to and including the term in x^2. **[4]**

(b) State the set of values of x for which this expansion is valid. **[1]**

(c) When $\dfrac{a+x}{(3-2x)^2}$ is expanded in ascending powers of x, the coefficient of x is zero.

Determine the value of the constant a. **[2]**

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4 **(a)** Show that the equation $2\cot^2 x - 9\operatorname{cosec} x - 3 = 0$ can be expressed in the form

$5\sin^2 x + 9\sin x - 2 = 0$. **[3]**

(b) **(i)** **In this question you must show detailed reasoning.**

Hence solve, for $0 < \theta < \pi$,

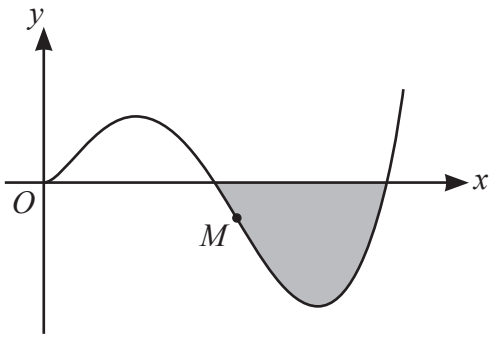
$2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3 = 0$.

Give your answers correct to **3** decimal places. **[4]**

The small angle approximation for $\sin 2\theta$ is used to find an approximation for the smallest positive solution of the equation $2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3 = 0$.

(ii) Show that this approximate solution is accurate to **2** decimal places. **[2]**

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The diagram shows the curve with equation $y = (x^3 - 2x^2)\ln x$. The curve has a point of inflection at the point M.

(a) **(i)** Show that the x-coordinate of M satisfies the equation

$$x = \frac{6 + (4 - 6x)\ln x}{5}.$$ **[5]**

(ii) Use an iterative formula, based on the equation in part **(a)(i)**, to determine the x-coordinate of M correct to **2** decimal places. Use an initial value of 1.1 and show the result of each step of the iterative process. **[2]**

(b) Determine the exact area of the shaded region, giving your answer in the form $p\ln q - r$, where p and r are positive rational numbers and q is a positive integer. **[6]**

6 The curve C is defined, for $0 \leqslant t < 2\pi$, by the parametric equations

$x = 4k + k\sin t, \quad y = 2 + 4\cos t,$

where k is a constant.

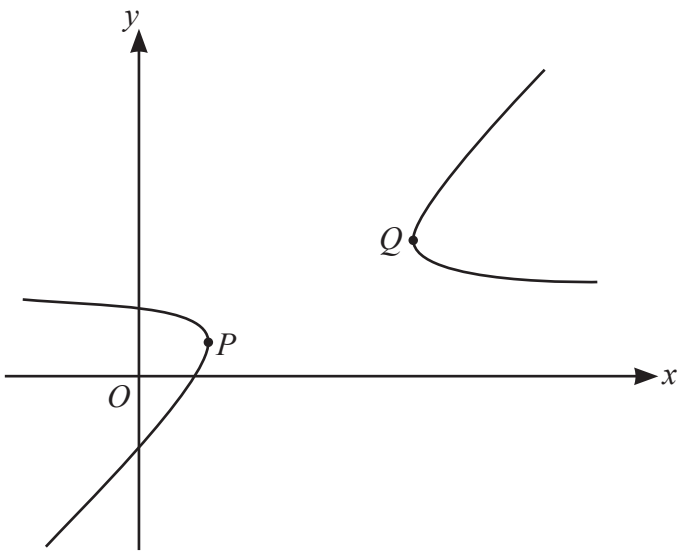
(a) Find a cartesian equation for C. You do **not** need to simplify your answer. **[2]**

You are given that C is a circle.

(b) (i) Determine the radius of C. **[2]**

(ii) Find the possible coordinates for the centre of C. **[2]**

7



The diagram shows the curve $5x - 2xy + 2y^2 - k = 0$, where k is a positive integer.

At the points P and Q on the curve, the tangents to the curve are parallel to the y-axis.

Given that the difference in the y-coordinates of P and Q is 3, determine the x-coordinates of P and Q. **[7]**

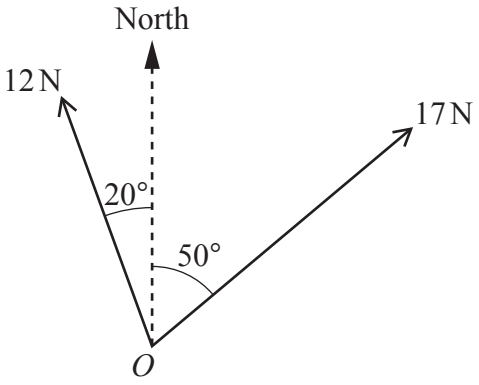
Section B
Mechanics

8 A particle P is moving with constant acceleration $(-5\mathbf{i}+2\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t = 0$ seconds, P is at the origin and has velocity $(\mathbf{i}+3\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find, in terms of **i** and **j**, the displacement of P at time $t = 2$ seconds. **[2]**

(b) Determine the speed of P at time $t = 2$ seconds. **[4]**

9



Two horizontal forces of magnitudes 17 N and 12 N act at a point O along bearings of 050° and 340° respectively (see diagram).

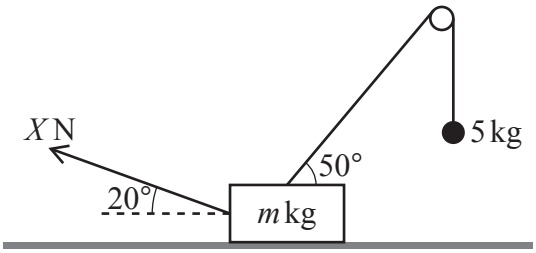
(a) Determine the magnitude and bearing of the resultant force. **[6]**

A third horizontal force **F** is now applied at O. The three forces are in equilibrium.

(b) State the magnitude of **F** and give the bearing along which it acts. **[2]**

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A block of mass m kg is on smooth horizontal ground with one end of a light inextensible rope attached to its upper surface. The other end of the rope is attached to an object of mass 5 kg. The rope passes over a small smooth pulley, and the object hangs vertically below the pulley. The part of the rope between the block and the pulley makes an angle of 50° with the horizontal. A force of magnitude X N acts on the block at an angle of 20° above the horizontal in the vertical plane containing the rope (see diagram).

You are given that the block is in equilibrium.

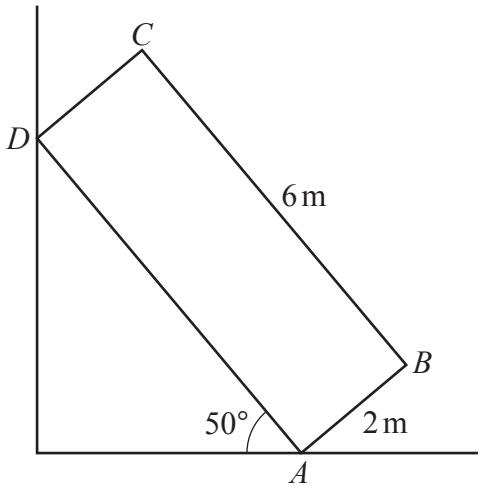
(a) Determine the value of X. **[3]**

You are also given that the magnitude of the contact force exerted by the ground on the block is 147 N.

(b) Determine the value of m. **[3]**

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A uniform rectangular lamina $ABCD$ has a mass of 0.5 kg. The length of AB is 2 m, and the length of BC is 6 m. The lamina is in limiting equilibrium with corner A in contact with rough horizontal ground and corner D in contact with a smooth vertical wall. The lamina rests in a vertical plane that is perpendicular to the wall, with AD inclined at 50° to the horizontal (see diagram).

(a) By taking moments, show that the magnitude of the normal contact force between the lamina and the wall is 1.24 N, correct to **3** significant figures. **[4]**

(b) Determine the coefficient of friction between the lamina and the ground. **[3]**

12 A particle P moves in a straight line. The velocity $v\,\text{m}\,\text{s}^{-1}$ of P at time t seconds is given by

$v = \frac{1}{12}kt(t-3)$ for $0 \leqslant t \leqslant 6$,

$v = \dfrac{54k}{t^2}$ for $6 \leqslant t \leqslant 9$,

where k is a positive constant.

(a) Sketch, on the axes in the Printed Answer Booklet, the velocity-time graph for P for values of t from 0 to 9. **[3]**

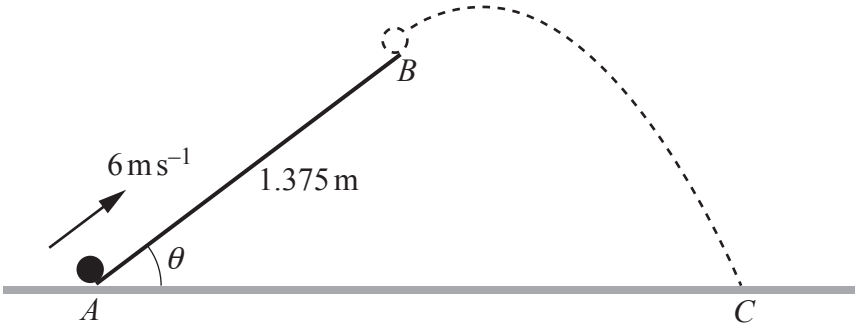
(b) State the value of t in the interval $0 \leqslant t \leqslant 9$ when the acceleration of P is zero. **[1]**

(c) **In this question you must show detailed reasoning.**

You are given that the total distance travelled by P in the interval $0 \leqslant t \leqslant 9$ is 84 m.

Find the value of k. **[6]**

13



The points A and B are the lower and upper ends, respectively, of a line of greatest slope on a plane inclined at an angle θ to the horizontal, where $\sin\theta = 0.6$ and $AB = 1.375\,\text{m}$ (see diagram).

A particle P is projected up the plane with speed $6\,\text{m s}^{-1}$ from A towards B.

The plane at A is fixed to the ground which is horizontal.

The surface of the plane is rough and the coefficient of friction between P and the plane is 0.25.

(a) Show that the speed of P at B is $3.8\,\text{m s}^{-1}$. **[6]**

The particle leaves the slope at B and moves freely under gravity.

The particle first lands at a point C on the horizontal ground. The time taken for P to travel from A to C is T seconds.

(b) Determine the value of T. **[6]**

END OF QUESTION PAPER

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